English and LaTeX for Mathematicians

Melchior Grützmann

melchiorG.freehosting.com/english
Department of Mathematics

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Outline

Last week: Giving Presentations

This week: posters in XƎLaTeX

2. Structure of a paper

Next: More about writing math

Structuring research papers

0. Abstract
1. Introduction
2. Preliminaries
3. ⟨Main theorem⟩
4. further material (optional)
99. References

Formulating definitions and theorems

- full sentences,
- all preliminaries,
- check whether claim correct

Example: Definition

Definition

A Lie algebra is a vector space $\mathfrak{g}$ *together with a bilinear skew-symmetric operation* $[.,.]\colon \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$ *subject to the Jacobi identity*

$$[X,[Y,Z]] + [Y,[Z,X]] + [Z,[X,Y]] = 0$$

for all $X, Y, Z \in \mathfrak{g}$.

Proposition

A vector subspace $\mathfrak{h} \subset \mathfrak{g}$ *of a Lie algebra* $(\mathfrak{g}, [.,.])$ *is a Lie subalgebra iff the bracket closes on* $\mathfrak{h}$, *i.e.* $[\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}$.

Example: Theorem

Definition

A Lie algebra $\mathfrak{g}$ is said to be semi-simple iff $[\mathfrak{g}, \mathfrak{g}] = \mathfrak{g}$.
A vector subspace $I \subset \mathfrak{g}$ is called ideal iff $[I, \mathfrak{g}] \subset I$.
A Lie algebra is said to be simple iff it has no (nontrivial) ideals.

Theorem (Krull–Schmidt)

A finite dimensional semi-simple Lie algebra is the direct sum of simple Lie subalgebras.

Formulating examples

Example (Lie algebras)

1. The real space $\mathbb{R}^3$ together with the vector product $\times$, defined as $\mathrm{e}_1 \times \mathrm{e}_2 = \mathrm{e}_3$, $\mathrm{e}_2 \times \mathrm{e}_3 = \mathrm{e}_1$, $\mathrm{e}_3 \times \mathrm{e}_1 = \mathrm{e}_2$ and extended linearly and skew-symmetrically forms a Lie algebra. This is denoted as $\mathfrak{so}(3)$.
2. Given the $n \times n$ matrices $\mathrm{Mat}_{n\times n}(\mathbb{R})$, then

$$[A, B] := A \circ B - B \circ A$$

   for $A, B \in \mathrm{Mat}_{n\times n}(\mathbb{R})$ and $\circ$ the matrix multiplication, forms a Lie algebra denoted as $\mathfrak{gl}_n(\mathbb{R})$.
3. The classical Lie algebras $\mathfrak{o}(n) = \mathfrak{so}(n)$, $\mathfrak{u}(n)$, $\mathfrak{su}(n)$, $\mathfrak{sp}(n)$, $\mathfrak{sl}_n(\mathbb{R})$ are ...

Formulating proofs

1. State what you need to prove,
2. prove it,
3. say that you have proved it.

- Make it read smoothly.

Example: Proof

Proof.

We want to show that $\mathfrak{gl}_n(\mathbb{R})$ is indeed a Lie algebra. The bracket is bilinear, because matrix multiplication is. By inspection the bracket also is skew-symmetric. It remains to show the Jacobi identity. We will start with the well-known fact that matrix multiplication is associative, i.e. $(A \circ B) \circ C = A \circ (B \circ C)$ for every $A, B, C \in \mathfrak{gl}_n(\mathbb{R})$. We can therefore avoid parentheses in the following computation

$$\begin{aligned}[A,[B,C]] &= [A, B \circ C - C \circ B] \\ &= A \circ B \circ C - A \circ C \circ B - B \circ C \circ A + C \circ B \circ A.\end{aligned}$$

Cyclic permutation of A, B, and C leads to

$$\begin{aligned}[B,[C,A]] &= B \circ C \circ A - B \circ A \circ C - C \circ A \circ B + A \circ C \circ B, \\ [C,[A,B]] &= C \circ A \circ B - C \circ B \circ A - A \circ B \circ C + B \circ A \circ C.\end{aligned}$$

Adding up the three equations leads to the Jacobi identity. This completes the proof. □

Putting things together

Book

1. Name some chapters,
2. write them one by one, going back to add / change whenever necessary,
3. proof read and rearrange,
4. write the introduction

Article

1. Start with the statement of the main theorem(s),
2. state a main definition,
3. write the proof and add definitions and lemmas as required,
4. give examples
5. proof-read
6. write the introduction

Formulating an introduction

1. Search for more related literature;
2. Explain where the main definition is used in your area;
3. avoid formulas – this is not the main body of the paper;
4. Explain the meaning of your main theorem, implications, interpretation;
5. Relate to other people's research (earlier results, conjectures, the big picture);

optional give outlook on further related questions.

7. contents of each section

Style

Things to avoid
- emotional adjectives: excellent, good, nice, bad, awful, important, . . . ;
- long complicated sentences;
- imprecise formulation;
- grammar mistakes

Helpful
+ use math terms properly;
+ consistent use of symbols;
+ reused examples;

Where to put conclusions

Out of 10000 active mathematicians so many read the following of an important paper:

3000	title,
1000	abstract,
300	introduction,
250	statement of main theorem,
250	some preliminaries,
50	proof

Therefore, in most math papers conclusions are put in the middle / at the end of the introduction or as corollaries after the main theorem.

Writing an abstract

1. Summarize the main theorem(s) in one sentence each.
2. avoid math symbols;
3. only 1–2 short paragraphs

Choosing a title

- informative,
- short,
- honest

– "On Courant algebroids"

\+ "Structure of regular Courant algebroids"

Further reading

- math literature in your field
- good math lectures
- P.R. Halmos: *How to write math*. in L'Enseignement Mathématique, vol. 16 **(1970)**.
  www.math.uga.edu/~azoff/courses/halmos.pdf

Next: More about writing math

- good / bad formulations
- some mathematical notions