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Department: _____

Sample Midterm Exam Calculus for Engineers I

本人知晓我校考场规则和违纪处分条例的有关规定，保证遵守考场规则，诚实做人。

Cheating in the exam is punishable by expulsion from class!

This exam consists of 9 questions and has 100 points in total. The exam duration is 90min. You may not use a calculator or prewritten notes in this exam.¹ Please turn off and put away your cell-phone.

In order to obtain full points for a question, all work must be shown and readable.

1. Limits & Continuity

1. (12 points) Compute the following limits! (Justify your answer.)

(a) (3 points)

$$\lim_{x \rightarrow -1} \frac{x^4 - x^2}{2x^3 - x^2 + x + 4}$$

(b) (3 points)

$$\lim_{t \rightarrow -\infty} \sin \arctan \frac{t^2}{t+1}$$

(c) (3 points)

$$\lim_{z \rightarrow 0^-} e^{1/\sin z}$$

¹An alphabetically sorted vocable list or dictionary without annotations is permitted.

(d) (3 points)

$$\lim_{x \rightarrow \infty} \left(\frac{\sin x}{x^\alpha} \right)$$

where $\alpha > 0$.

2. (10 points) Where is the following function continuous? (Justify your answer.)

$$g(z) = \frac{1}{\sin(\tan z)}$$

3. (10 points) Assume that the (annual average) temperature changes continuously along the equator. Show that there are two opposite points on the equator that have the same (annual average) temperature.

0. Functions & Models

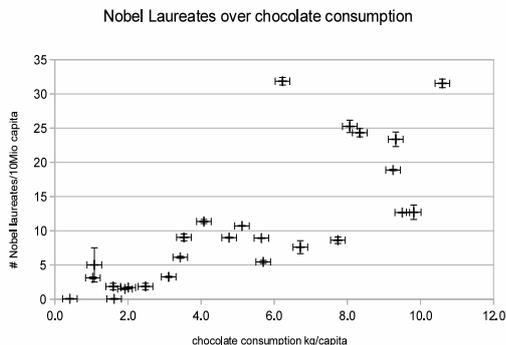
4. (10 points) Consider the distribution of Nobel prices per 100'000'000 capita together with the chocolate consumption

country	China	India	USA	Pakistan	Germany	France	UK
N # Nobel prices ¹	8	9	336	1	103	59	119
n Nobel prices/ 10^7 capita	0.060	0.075	10.71	0.05	12.7	9.0	18.9
chocolate consumption ϕ [kg/capita]	0.4		5.1		9.5	4.8	9.3
$\ln n$	-2.81	-2.59	2.37	-3.00	2.54	2.20	2.94
$\ln \phi$	-0.92		1.63		2.25	1.57	2.23
$\ln(\phi + 1)$	0.06		1.81		2.35	1.76	2.33

source(s): theobroma-cacao.de, shockmd.com, theCNNfreedomProject, 2010

¹since 1901

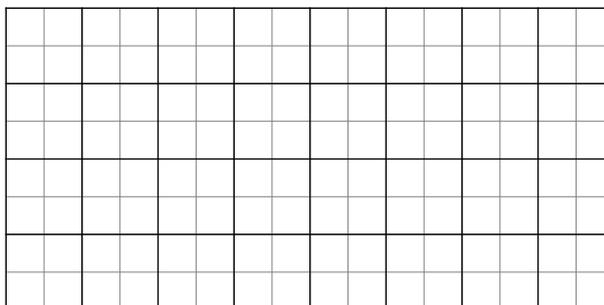
Plotted directly, they look as follows



(a) (2 points) Draw a trend curve for the data points and decide how to model the data. Possible models are:

- (1) $n = m\phi + b$,
- (2) $n = Ae^{c\phi}$,
- (3) $n = k \ln(\phi + 1) + C$,
- (4) $n = A'\phi^n$

(b) (6 points) According to your model choose what to plot on the horizontal and vertical axes such that the points should form a straight line. Draw the (at least 5) data points into the following grid together with a regression line. Read off the parameters of the line.



Please make sure to label and scale the axes.² If you obtain fractions, e.g. $\frac{3.14}{0.79}$, you may leave them in this form.

(c) (2 points) According to your model and regression data, give a formula that estimates the number of Nobel prices for Switzerland (population $8 \cdot 10^6$, chocolate consumption $\phi = 10.6\text{kg/yr} \cdot \text{cap}$). Give another formula that estimates how much chocolate must be eaten per year and capita to obtain 20 Nobel prices (with a population of 10^7 over similar 109 years). (No numerical result needed.)

²The axes don't have to start at 0, but should be linear.

2. Derivatives

5. (18 points) Compute the derivative of the following functions!

(a) (5 points)

$$f(x) = x^{\sin x / \ln x}$$

(b) (5 points)

$$g(y) = \arctan \frac{y+1}{y-1}$$

(c) (5 points)

$$h(z) = \frac{\frac{1}{2}z - 2}{(z-2)^2(2z-3)^3}$$

(d) (3 points) Where does the last function $h(z)$ have a vertical asymptote?

6. (10 points) Compute the leading coefficient of the 10th derivative:

$$e^{x^2} \frac{d^{10}}{dx^{10}} e^{-x^2+1}$$

Hint: You can either find a general formula or differentiate 10 times.

7. (10 points) For which constants $a \in \mathbb{R}$ does the function $x(t) = Ce^{at+b}$ fulfill the equation

$$\ddot{x} = \frac{k}{m}x(t) \quad \text{for all } t \in \mathbb{R} ?$$

where $b, C \in \mathbb{R}$ are real constants, $C \neq 0$, and $\ddot{x} = \frac{d^2x}{dt^2}$ is the second derivative.

8. (10 points) Where is the following parametrized curve not differentiable?

$$x = t^4 - t^2, \quad y = t^3 - t$$

Hint: With (horizontal) vertical tangent line the curve would still be differentiable.

9. (10 points) Compute the tangent line to

$$|x|^\alpha + |y|^\alpha = 1 \quad \text{at the point with } \frac{x}{y} = -1!$$

Where $\alpha > 0$ is a constant.