

# Abstract algebra: Homework 6

Northwestern Polytechnic University

Due on Monday, Nov. 19<sup>th</sup>

## 1.10 Small groups

**Exercise 1.10.1** (2P). To which group of order 12 is  $C_2 \oplus D_3$  isomorphic?

Non-abelian groups in the following exercise should be specified by (easy) presentations.

**Exercise 1.10.2** (8P). Find all groups of order

- a. 51,
- b. 21,
- e. 57,
- f. 93.

## 1.11 General linear group & Group representations

**Exercise 1.11.1** (5P). Given a finite subgroup of  $G \subset \text{GL}(V)$  a real (or complex) vector space, show that

- a. every  $g \in G$  has determinant  $\det g \in \Omega_*$  the group of roots of unity 1,
- b. give an example of an element in  $g \in \text{GL}_2(\mathbb{R})$  that has finite order, but not determinant 1,
- c.  $G$  is isomorphic to a subgroup of  $\text{O}(V)$  (or  $\text{U}(V)$  respectively).

**Exercise 1.11.2** (5P). Decompose the (left)-regular representation of  $S_3 \times C_4$  into irreducible representations.