

# Abstract algebra: Homework 5

Northwestern Polytechnic University

Due on Monday, Nov. 12<sup>th</sup>

## 1.8 Symmetric groups

**Exercise 1.8.1** (2P).

- a. Show that  $S_n$  is generated by  $(12), (23), \dots, (n-1 n)$ .
- b. Show that  $S_n$  is generated by  $(12)$  and  $(12 \dots n)$ .

**Exercise 1.8.2** (1P).

- a. Show that  $S_4 \cong \langle a, b : a^4 = \text{id} = b^2 = (ba)^3 \rangle$ .

**Exercise 1.8.3** (2P). How many  $k$ -cycles are there in  $S_n$ ?

**Exercise 1.8.4** (3P). Consider the permutation  $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 5 & 6 & 4 & 2 & 8 & 3 & 1 \end{pmatrix}$

- a. Write  $\sigma$  as product of disjoint orbits. Determine its signum and its order.
- b. What is the order of the centralizer of  $\sigma$  in  $S_8$ ? What is the order of the conjugacy class of  $\sigma$ ?

**Exercise 1.8.5** (2P).

- a. List all conjugacy classes of  $S_5$  together with their orders.

Please turn the page.

## 1.9 Sylow theorem

**Exercise 1.9.1** (1P). Given a finite group whose order is divisible by a prime  $p$ . Show that there is a subgroup of order  $p$ .

**Exercise 1.9.2** (2P). Find the Sylow subgroups of

- a.  $S_4$  and
- b.  $S_5$ .

**Exercise 1.9.3** (3P). Find all groups of order

- a. 33,
- b. 35,
- c. 45.

**Exercise 1.9.4** (4P). Show that the following are not simple groups, i.e. that they have a non-trivial normal subgroup:

- a. A group of order 18;
- b. A group of order 30;